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Analytical solutions of fracture parameters for a centrally cracked Brazilian disk considering the loading friction

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ABSTRACT
A centrally cracked Brazilian disk (CCBD) specimen subjected to a pair of diametral compressive forces has been widely used to study mixed-mode I and II fractures of brittle and quasi-brittle materials. Reasons for using the CCBD are mainly due to its capability to introduce different mode mixities from pure mode I to pure mode II, the existence of closed-form solutions for fracture parameters, and the simple setup of compressive test. In addition to the diametrical concentrated force loading, the partially distributed pressure loading is also an important loading condition for CCBD specimen tests. Using the weight function method, analytical solutions of stress intensity factors and T stress considering the tangential loading friction for a CCBD specimen that is subjected to four typical partially distributed loads were derived, and effects of the boundary friction and load distribution angle on the fracture parameters were also explored. The results obtained are as follows: (1) For short cracks, geometric parameters YI, YII, and T* of pure mode I and II fractures decrease with an increase in the friction coefficient and load distribution angle. However, for long cracks, an increase in the friction coefficient causes an increase in pure mode-I YI, and an increase in the load distribution angle causes an increase in pure mode-II T*; (2) The influence of the load distribution angle on the fracture parameters is the most significant when the distributed pressure follows a constant function form, while it is the least significant for the case of quartic polynomial pressure; (3) The critical loading angle for pure mode II fractures decreases with an increase in the load distribution angle for short cracks, whereas it increases for long cracks. When the load distribution angle is fixed, an increase in friction can raise the critical loading angle for pure mode II fractures. These results have further improved the research of fracture parameters in CCBD specimens.

KEYWORDS:
centrally cracked Brazilian disk, fracture parameters, boundary friction, distributed load, analytical solutions

1. Introduction

The fracture parameters of a centrally cracked Brazilian disc (CCBD) include the stress intensity factors of type I and type II and T stress [1- 5]. T stress is the constant term in the William expansion of the stress field at the crack tip. A large number of studies have shown that the influence of T stress cannot be ignored due to the relatively large size of the fracture process zone of brittle materials such as rocks [6- 8].

Since CCBD specimens can undergo any type I/II composite fracture under radial compression and their fracture parameters have analytical solutions, they are widely used in the study of type I/II composite fracture of brittle and quasi-brittle materials such as glass [9], ceramics [10], and rocks [11- 13]. Awaji and Sato [14] first proposed the use of CCBD specimens to measure the type I and type II fracture toughness of rocks. [1] first derived the type I and II fracture toughness of CCBD specimens using the boundary integral equation. However, [1] only gave the calculation coefficients of the first five terms of the analytical solution of the stress intensity factor series for the six cases of relative crack length β = 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. The unknown coefficients of other cases need to be obtained by numerical integration, which is inconvenient to apply. Fett [2], [3] proposed a weight function related to the stress intensity factors of CCBD specimens I and II and T stress, and then [4] used the weight function method to derive the full explicit series solution of the stress intensity factors of CCBD specimens I and II under concentrated load and uniform load, and explored the influence of the uniform load distribution angle on the stress intensity factor. Dorogoy and Banks-Sills [15] used a finite difference model to study the influence of the contact friction of the center crack on the stress intensity factors of I and II. [16] used the finite element method to calculate the type I and II stress intensity factors and T stress of CCBD specimens under concentrated load. Considering that most actual engineering materials are under confining pressure, [17], [18], [19] used the weight function method and superposition principle to derive the analytical solution of the fracture parameters of CCBD specimens under the combined action of radial concentrated load and confining pressure, and based on this analytical solution, discussed the effect of confining pressure on stress intensity factor and T stress. [20] used the interaction integral method based on extended finite element to solve the type I and II stress intensity factors and T stress of CCBD specimens. [21] used the weight function method to propose an analytical solution for T stress of CCBD specimens under uniform load. [22] used the interaction integral method to study the effect of confining pressure on stress intensity factor and T stress of CCBD specimens. The results show that the calculation results of the interaction integral method are very consistent with the results of the weight function method. [23] used the weight function method to derive the analytical solution of the stress intensity factor of CCBD specimens under radial concentrated load and uniform load with crack surface friction, and analyzed the influence of crack surface friction on the CCBD stress intensity factor. [24] used the weight function method to propose a calculation formula for the stress intensity factor of CCBD specimens under the combined action of confining pressure and radial concentrated load with crack surface friction, and theoretically analyzed the influence of confining pressure, radial load and crack surface friction on the CCBD stress intensity factor.

In the CCBD fracture test, in addition to the use of flat indenter loading, arc indenter loading is also a common means of applying external loads. [4], [21], [23] [25] all considered uniform load loading. This stress boundary is a simplification of arc indenter loading. However, on the one hand, the actual distribution of the contact force between the specimen and the arc indenter is unknown. Previous researchers have assumed that the distribution of the contact force is uniform distribution [26- 28], elliptical distribution [29- 31], cosine distribution [31], [32] quadratic function distribution [31- 33] and quartic function distribution [31]. Compared with uniform distribution, [30] pointed out that the distribution of contact force is more likely to be non-uniform. On the other hand, few scholars have considered the friction between the arc indenter and the specimen and explored the influence of end friction on fracture parameters. Recently, [34] The analytical solution of the type I stress intensity factor of the CCBD specimen under pure type I loading considering the friction at the loading end is derived by using the complex variable function theory and the weight function method. However, it does not focus on the analytical solution of the fracture parameters of the CCBD specimen under type I/II composite loading conditions. In view of this, this paper first uses the weight function method to derive the analytical solution of the fracture parameters of the CCBD specimen under any type I/II composite fracture mode under four forms of distributed loads (uniform function, elliptic function, quadratic function and quartic function, because the cosine function is similar to the quadratic function, it is not considered in this paper) considering the end friction. Then, based on the analytical solution, the influence of the end friction and the contact load distribution angle on the fracture parameters is discussed, in order to further improve the research on the fracture parameters of CCBD specimens.

2. Conclusions

Based on the weighted function method of fracture mechanics, the analytical solutions of the stress intensity factors of type I and type II and the T stress of CCBD specimens considering end friction are derived when the distributed load is in the form of uniform function, elliptic function, quadratic function and quartic function. On this basis, the influence of contact load distribution angle and friction coefficient on fracture parameters is explored. The specific conclusions are as follows:

(1) When the relative length β of the central crack is small, the geometric parameters of pure type I and pure type II fracture decrease approximately linearly with the increase of friction coefficient (for T*, it refers to its absolute value); when β is large, the increase of friction coefficient can make pure type I YI increase approximately linearly, and other geometric parameters still decrease.

(2) When β is small, the increase of contact load distribution angle can reduce the geometric parameters of pure type I and pure type II fracture; and when β is large, the increase of load distribution angle can increase pure type II T*.

(3) When the contact load distribution form is a constant function, the load distribution angle has a great influence on the geometric parameters. The influence of YI, YII and T* is the most significant, followed by elliptic function and quadratic function, while the influence of load distribution angle on geometric parameters under quartic function is relatively small.

(4) When β is small, the pure type II loading angle decreases with the increase of load distribution angle; however, when β is large, it increases with the increase of load distribution angle. When the load distribution angle is constant, the increase of friction coefficient can increase the pure type II loading angle.

3. References

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